7.1 Integral as Net Change

The definite integral of a rate of change gives the net change.

$$\int_a^b f'(t)\,dt = f(b) - f(a)$$

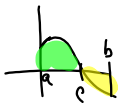
$X(t)$ = position $v(t)$ = velocity

displacement $\int_a^b v(t)\,dt = X(b) - X(a)$ final position − initial position

final position $X(b) = \int_a^b v(t)\,dt + X(a)$

total distance

(total area) $\int_a^b |v(t)|\,dt$




$\int_a^c v(t)\,dt + \left| \int_c^b v(t)\,dt \right|$

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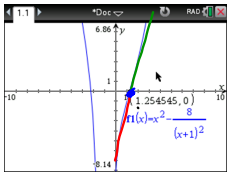
The velocity of a particle moving along the x-axis is given by:

$$v(t) = t^2 - \frac{8}{(t+1)^2}$$

a) Describe the motion left, right, at rest

b) The initial position of the particle is x(0)=9, what is the particle's position at t=1? at t=5?

c) Find the total distance traveled from t=0 to t=5.




a) left $0 \le t < 1.255$

at rest at $t = 1.255$

right $t > 1.255$

b) $\int_0^1 v(t)\,dt + 9 = 5.33$

$\int_0^5 v(t)\,dt + 9 = 44$

c) $\left| \int_0^{1.255} v(t)\,dt \right| + \int_{1.255}^5 v(t)\,dt = 42.6$

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Integral of a rate of change gives the total accumulation.

Potato Consumption - From 1970 to 1980 the rate of potato consumption was

$$C(t) = 2.2 + 1.1^t$$

millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1973?

$\int_2^4 2.2 + 1.1^t\,dt = 7.066$ Mbu

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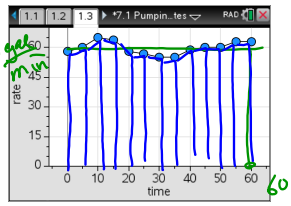
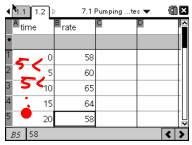
A pump connected to a generator operates at a varying rate shown in the table. How many gallons were pumped during the hour?




60 min $\frac{h}{2}\left[y_0 + 2y_1 + 2y_2 \ldots y_n\right]$

[sum(rate ÷2) − 58 − 63] · 5/2

3582.5 gal

60 gal/min · 60 min

3600 gal (area)

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Work done by a constant force: W = F*d

Work done by a variable force: $W = \int_a^b F(x)dx$

It takes a force of 10N to stretch a spring 2m beyond its natural length. How much work is done in stretching the spring 4m from its natural length?

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